A2CC Midterm Review Sheet

Midterm Exam: Wednesday, January 24th 10:00 – 11:30 in the Cafeteria

Exam Format: Tuesday $11^{30}-12^{30}$ question/answer

Part 1: 17 multiple choice questions (no partial credit)
Part 2: 9 free response questions (partial credit)

This review sheet should not be your only study guide. Please be sure to go over your old exams, homework and notes to fully prepare for the midterm.

Exponential Equations

1. Solve each of the following:
   (a) $8^x = 2^{x+6}$
   (b) $4^{2x-3} = \frac{1}{16}$
   (c) $x^{\frac{3}{2}} = 64$
   (d) $125^{-2x} = 25^{x+1}$
   (e) $a^{\frac{3}{5}} - 2 = 25$
   (f) $3(2m+3)^{\frac{2}{3}} + 2 = 77$

Rational Expressions

2. Simplify: $\frac{6x^2+12x}{x^3-5x^2-14x}$

3. Simplify: $\frac{x^2-2x-24}{x^2-16}$

4. Find the value(s) of x for which the fraction is undefined:
   (a) $\frac{12}{x-2}$
   (b) $\frac{23}{6x}$
   (c) $\frac{x^2-25}{x^2+6x+8}$

In 5 – 8, perform the indicated operations and express answers in simplest form.

5. $\frac{3}{x+4}+\frac{2}{x}$

6. $\frac{5x^2-5x-60}{4x^2-x}\cdot\frac{x^2-3x-10}{x-4}\div\frac{x^2-2x-15}{8x^2-2x}$

7. $\frac{2x^2-16}{x^2-4}-\frac{x+4}{x+2}$

8. $\frac{x-1}{x^2+3x+2}+\frac{x}{x+1}$

9. Solve for x : $\frac{1}{6x}+\frac{8}{x}=\frac{x}{6}$

10. Solve for x: $\frac{4n+3}{n-6}+\frac{n-4}{6-n}=\frac{44}{2n-12}$

11. Simplify each: (a) $\frac{1+\frac{2}{a}}{\frac{2}{a}-\frac{a}{2}}$ (b) $\frac{x-\frac{9}{x}}{1+\frac{3}{x}}$ (c) $\frac{1-\frac{1}{16x^2}}{1-\frac{1}{4x}}$ (d) $\frac{\frac{c}{2}-\frac{2}{c}}{1+\frac{c}{2}}$

Radicals

12. Simplify: $\frac{\sqrt{900}}{\sqrt{20}}$

13. $2\sqrt{48}+2\sqrt{12}$

14. $\sqrt{49a^2b^4}-\sqrt{16a^2b^4}+\sqrt{8a^2c}$

15. Simplify: $\frac{3}{6-5\sqrt{2}}$

16. Solve: $\sqrt{3x+6}-2=7$

17. Solve: $2\sqrt{2x-6}+8=4$

Complex Numbers

18. Simplify: $\sqrt{-45x^4y^7}$

19. $5\sqrt{-18}+\sqrt{-50}-\sqrt{-75}$

20. Find the value of i^{53}

21. $(2-5i)(6+7i)$

Factoring

22. Factor each of the following completely.
    (a) $3x^2 + 5x - 2$
    (b) $16x^4 - y^8$
    (c) $2x^2 - 10x - 28$
    (d) $x^3 + 3x^2 - 4x - 12$
    (e) $8x^3 + 125$

Quadratics

23. Find all roots of the equation: $2x^2 - 3x = 2$

24. Solve by completing the square: $3x^2 = 6x - 15$

25. Describe the roots of the following quadratic equations:
    (a) $2x^2 - 3x + 4 = 0$
    (b) $\frac{1}{3}x^2 - x = 6$
    (c) $x^2 - 6x = -9$

Inequalities

Solve each inequality and express the solution set in set builder notation.

26. $x^2 - x > 6$

27. $\frac{1}{x} < 1$

Solve each inequality and express the solution set in interval notation.

27. $x^2 - 7x \le x$

28. $\frac{9}{x-4} \ge -6$

Solving Higher Degree Polynomials

Solve each of the following.

29. $x^3 + 3x^2 - 9x = 27$

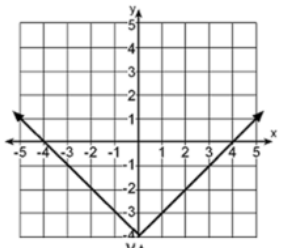
30. $x^4 - 8x^2 + 16 = 0$

31. $(x^2 + 5x - 7)(x + 3) = 0$

Functions

32. Give the domain and range for each relation. Then tell whether the relation is a function.

(a)



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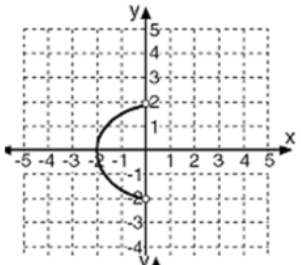

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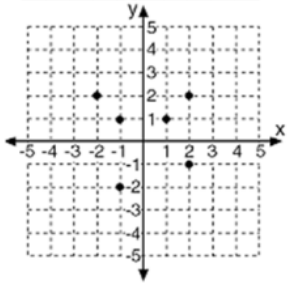
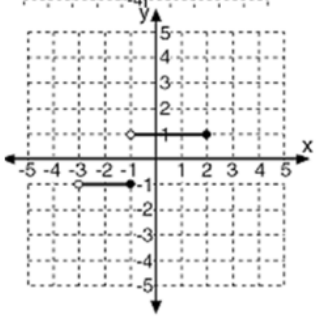


(d)



33. Given $f(x) = 4 - x^2$, evaluate $f(-2)$.

In 34 – 39, determine whether or not the following relations are functions. *If they are functions:*
(a) determine whether or not they are one – to –one

34. $3x = -10 + 4y$

35. $x = y^2 - 2y - 24$

36. $x = 5$

37. $y = 10$

38. $y = |2x - 3|$

39. $y = x^2 - x - 6$

40. Determine the domain of each of the following functions.

(a) $f(x) = \dfrac{x+2}{x^2 - x - 20}$

(b) $g(x) = \sqrt{2x - 5}$

(c) $h(x) = \dfrac{1}{\sqrt{3x - 15}}$

41. Given $f(x) = 2\sqrt{x+3}$ and $g(x) = -3x + 1$, find each value:

(a) $f(g(1))$
(b) $g(f(1))$
(c) $g(f(6))$

42. Given $f(x)=4x+3$ and $g(x)=\frac{x}{x+3}$, find:
(a) $f(g(x))$
(b) $g(f(x))$

43. Given $h(x)=-x^2-2$ and domain $-3\le x\le 3$, find the largest element in the range.

44. Given $f(x)=\frac{7-8x}{3}$. Is $f(x)$ one to one? Explain your answer.

45. Determine the domain and range of $f(x)=\{(1,2),(3,4),(5,6),(7,8)\}$. Find the inverse of $f(x)$. Is $f(x)$ one to one?

Laws of Exponents

46. Simplify each expression and write the answer using only positive exponents:

(a) $\frac{2x^{-2}y^{-2}}{4y^{-5}}$ (b) $\frac{3x^{-4}y^{5}}{\left(2x^{3}y^{-7}\right)^{-2}}$

47. Rewrite the expression $x^{-\frac{2}{5}}$ as an equivalent expression in radical form.

Polynomials

48. Write $\frac{2x^3-x^2-6x-1}{x+1}$ in the form $q(x)+\frac{r}{x+1}$, where q(x) is a polynomial and r is a constant.

49. Is (x+2) a factor of $3x^4+7x^3+3x^2-x-4$. Explain your answer.

50. What is the remainder when $6x^5+21x^4-14x^3-8x^2+x-6$ is divided by $(x+4)$?

51. What is the complete factorization of $4x^4-x^3-4x^2+1x$?

52. What is the complete solution set of $P(x)=x^6+2x^4-16x^2-32$?

53. Sketch $f(x)=(x+3)^2(x-5)$ (be sure to label all x and y intercepts)

Circles

54. Determine the center and radius of the circle whose equation is $x^2+y^2+8x+25=6y+15$

Probability

55. The following table shows the results of a survey of people in terms of what type of breakfast they prefer. Based on the table, what is the probability that a person picked at random is over 40 and eats eggs for breakfast?

	Eats Cereal	Eats Eggs
40 and under	23	17
Over 40	21	29

56. If a standard six sided die is rolled once, what is the probability that the number rolled is either an even or a multiple of 3?

57. Prime numbers are positive integers that are only divisible by 1 and themselves, i.e. the set $\{2, 3, 5, 7, ...\}$. If a random number is generated from 1 to 20, what is the probability that it is *not* prime?

58. Of all the tourists who visit Florida, 38% of them will visit an amusement park and 54% will visit a beach. If 22% will visit both an amusement park and a beach, then what percent will visit either a park or a beach?

59. If a restaurant is chosen at random in Rhinebeck then there is an 84% chance that it is open on Sunday and a 42% chance that it is open on Monday. If there is a 96% chance it is open on either Sunday or Monday, what is the probability that it is open both days?

60. A single standard six-sided die is rolled. What is the probability the roll is a multiple of three given that it is an even number?

61. The probability on any given work day that Kirk gets less than five hours of sleep the night before and doesn't shave is 0.65. If there is a 0.80 probability on any given day that he doesn't shave and a 0.70 probability he gets less than five hours of sleep, then what is the probability he doesn't shave given that he got less than five hours of sleep?

62. A die is rolled three times and a curious pattern emerges. On the first roll, the number is greater than 3. On the second roll, the number is greater than 4, and on the third roll, the number is greater than 5. If all three rolls are independent, what is the probability that this occurs?

63. Algernon is running a science fair experiment where mice run through a maze with 4 turns. At each turn, the mouse can take a right or a left. A mouse will find an exit if they either take two rights followed by two lefts or a left followed by two rights and then a left again. Assuming that each turn is independent of all previous ones, what it the probability that a mouse will find an exit. Show how you arrived at your answer.

64. A school system did not use up all of its snow days and will get four of them back as vacation days, either in April or in May. A survey was done amongst the student body to determine the preference for which month to have the days off. The results are presented below arranged by class.

	April	May
9th Grade	166	64
10th Grade	160	96
11th Grade	124	117
12th Grade	88	132

(a) What percent of the students preferred having the days off in April? Round to the nearest percent.

(b) If a student from this survey was chosen at random, what is the probability they would be an upperclassman (11th or 12th) and preferred having days off in May?

(c) If a student is chosen at random, what is the probability that they are a 10th grader given that they preferred to have the days off in April?

(d) Is the preference for the month independent of the grade of the student? Explain how you made your determination.

65. In a survey of 500 high school students, 85% said they liked pizza while 68% said they liked hot dogs and 61% reported liking both. How many students in the survey reported liking neither pizza nor hot dogs? Show how you arrived at your answer.

1) a) $8^x = 2^{x+6}$

(*) If Bases are = Exponents are =

$(2^3)^x = 2^{x+6}$

$2^{3x} = 2^{x+6}$

$3x = x+6$

$-x \quad -x$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

b) $4^{2x-3} = \frac{1}{16}$

$4^{2x-3} = (16)^{-1}$

$4^{2x-3} = (4^2)^{-1}$

$2x-3 = -2$

$+3 \quad +3$

$\frac{2x}{2} = \frac{1}{2}$

$x = \frac{1}{2}$

c) $x^{\frac{3}{2}\left(\frac{2}{3}\right)} = 64^{\left(\frac{2}{3}\right)}$

$x = 16$

d) $125^{(-2x)} = 25^{x+1}$

(*) careful to Distribute

$25 = 5^2$

$125 = 5^3$

$(5^3)^{(-2x)} = (5^2)^{x+1}$

$5^{-6x} = 5^{2x+2}$

$-6x = 2x+2$

$-2x \quad -2x$

$\frac{-8x}{-8} = \frac{2}{-8}$

$x = -\frac{1}{4}$

(*) To Solve use Reverse PEMDAS (+2 first)

e) $a^{\frac{3}{5}} - 2 = 25$

$+2 \quad +2$

$a^{\frac{3}{5}\left(\frac{5}{3}\right)} = 27^{\left(\frac{5}{3}\right)}$

$a = 243$

f) $3(2m+3)^{\frac{2}{3}} + 2 = 77$

$-2 \quad -2$

$\frac{3(2m+3)^{\frac{2}{3}}}{3} = \frac{75}{3}$

Even add need ±

$(2m+3)^{\frac{2}{3}\left(\frac{3}{2}\right)} = 25^{\left(\frac{3}{2}\right)}$

(*) "OLD McDonald" PLUS & MINUS IS the rule Even over ODD* *original fraction

$2m+3 = \pm 125$

$2m+3 = 125$ or $2m+3 = -125$

$-3 \quad -3$ $\quad -3 \quad -3$

$\frac{2m}{2} = \frac{122}{2}$ $\quad \frac{2m}{2} = \frac{-128}{2}$

$m = 61$ $\quad m = -64$

2) ⊛Simplify = Factor (Factor First) Don't forget restrictions

$$\frac{6x^2+12x}{x^3-5x^2-14x} = \frac{6x(x+2)}{x(x^2-5x-14)} = \frac{6x(x+2)}{x(x-7)(x+2)}$$

restrictions:
$x \neq 7, -2, 0$

$$= \boxed{\frac{6}{x-7}}$$

3)

$$\frac{x^2-2x-24}{x^2-16} = \frac{(x-6)(x+4)}{(x+4)(x-4)} = \boxed{\frac{x-6}{x-4}}$$

rest:
$x \neq 4, -4$

4) a) $\frac{12}{x-2}$ $x-2=0$, $+2\ +2$, $x=2$ $\boxed{x=2}$

b) $\frac{23}{6x}$ $\frac{6x}{6}=\frac{0}{6}$ $\boxed{x=0}$

c) $\frac{x^2-25}{x^2+6x+8}$

$x^2+6x+8=0$
$(x+4)(x+2)=0$
$\boxed{x=-4 \quad x=-2}$

5) $\frac{(x)3}{(x)x+4} + \frac{2(x+4)}{x(x+4)}$

LCD: $x(x+4)$

$x \neq 0, -4$

$\frac{3x}{x(x+4)} + \frac{2x+8}{x(x+4)} = \boxed{\frac{5x+8}{x(x+4)}}$

6) $\frac{5(x^2-x-12)}{5x^2-5x-60} \cdot \frac{x^2-3x-10}{x^2-x} \cdot \frac{8x^2-2x}{x^2-2x-15}$

$\frac{5(x-4)(x+3)}{} \quad \frac{(x-5)(x+2)}{x(x-1)} \quad \frac{2x(4x-1)}{(x-5)(x+3)}$

$x \neq 0, \frac{1}{4}, 4, 5, -3$

$5(x+2)(2) = \boxed{10x+20}$

7) $\frac{2(x^2-8)}{2x^2-16} - \frac{(x+4)(x-2)}{x+2(x-2)}$

$\frac{2x^2-16}{x^2-4}$

$(x+2)(x-2)$

LCD: $(x+2)(x-2)$

$x \neq -2, 2$

$\frac{2x^2-16}{(x+2)(x-2)} - \frac{x^2+2x-8}{(x+2)(x-2)} = \frac{(x-4)(x+2)}{(x+2)(x-2)} = \boxed{\frac{x-4}{x-2}}$

8) $\frac{x-1}{x^2+3x+2} + \frac{x(x+2)}{x+1(x+2)}$

$(x+2)(x+1)$

LCD: $(x+2)(x+1)$

$x \neq -1, -2$

$\frac{x-1}{(x+2)(x+1)} + \frac{x(x+2)}{(x+1)(x+2)} = \frac{x-1+x^2+2x}{(x+2)(x+1)}$

$= \frac{x^2+3x-1}{(x+2)(x+1)}$

9) $(6x)\frac{1}{6x} + \frac{8}{x}(6x) = \frac{x}{6}(6x)$

LCD: $6x$

$x \neq 0$

$1 + 48 = x^2$

$49 = x^2$

$\boxed{\pm 7 = x}$

10) $\frac{1}{-4} = \frac{-1}{4}$

$$\frac{4n+3}{n-6} + \frac{n-4}{6-n} = \frac{22}{2n-12}$$

$(-1)(n-6)$ $\quad 2(n-6)$

LCD: $(n-6)$, $n \neq 6$

$$\frac{4n+3}{n-6} + \frac{-1(n-4)}{n-6} = \frac{22}{n-6}$$

← Same denominator, so solve the "Tops"

$$4n+3-n+4 = 22$$
$$3n+7 = 22$$
$$-7 \quad -7$$
$$\frac{3n}{3} = \frac{15}{3}$$
$$n = 5$$

11) a) LCD: $2a$, $a \neq 0$, $a \neq 2, -2$

$$\frac{(2a)\frac{1}{1} + \frac{2}{a}(2a)}{(2a)\frac{2}{a} - \frac{a}{2}(2a)} = \frac{2a+4}{4-a^2} = \frac{2(a+2)}{(2-a)(2+a)} = \boxed{\frac{2}{2-a}}$$

b) LCD: x, $x \neq 0$, $x \neq -3$

$$\frac{(x)\frac{x}{1} - \frac{9}{x}(x)}{(x)\frac{1}{1} + \frac{3}{x}(x)} = \frac{x^2-9}{x+3} = \frac{(x+3)(x-3)}{x+3} = \boxed{x-3}$$

c) LCD: $16x^2$, $x \neq 0$, $x \neq \frac{1}{4}$

$$\frac{(16x^2)\frac{1}{1} - \frac{1}{16x^2}(16x^2)}{(16x^2)\frac{1}{1} - \frac{1}{4x}(16x^2)^{4x}} = \frac{16x^2-1}{16x^2-4x} = \frac{(4x+1)(4x-1)}{4x(4x-1)}$$

$$\boxed{\frac{4x+1}{4x}}$$

d) LCD: $2c$, $c \neq 0$, $c \neq -2$

$$\frac{(2c)\frac{c}{2} - \frac{2}{c}(2c)}{(2c)\frac{1}{1} + \frac{c}{2}(2c)} = \frac{c^2-4}{2c+c^2} = \frac{(c+2)(c-2)}{c(2+c)}$$

$$\boxed{\frac{c-2}{c}}$$

12) $\frac{\sqrt{900}}{\sqrt{20}} = \sqrt{\frac{900}{20}} = \sqrt{45}$

$\sqrt{9}\ \sqrt{5}$

$\boxed{3\sqrt{5}}$

13) $2\sqrt{48} + 2\sqrt{12}$

$2\sqrt{16}\sqrt{3}$ $\quad$ $2\sqrt{4}\sqrt{3}$

$2 \cdot 4\sqrt{3}$ $\quad$ $2 \cdot 2\sqrt{3}$

$8\sqrt{3} + 4\sqrt{3}$

$\boxed{12\sqrt{3}}$

14) $\sqrt{49a^2b^4} - \sqrt{16a^2b^4} + \sqrt{8a^2c}$

$\sqrt{4a^2}\ \sqrt{2c}$

$7ab^2 - 4ab^2 + 2a\sqrt{2c}$

$\boxed{3ab^2 + 2a\sqrt{2c}}$

15) $\frac{3}{6-5\sqrt{2}} \cdot \frac{6+5\sqrt{2}}{6+5\sqrt{2}} = \frac{18+15\sqrt{2}}{36-25\sqrt{4}} = \frac{18+15\sqrt{2}}{36-25(2)}$

$= \frac{18+15\sqrt{2}}{36-50}$

$= \boxed{\frac{18+15\sqrt{2}}{-14}}$

16) $\sqrt{3x+6} - 2 = 7$

$+2 \quad +2$

$(\sqrt{3x+6})^2 = (9)^2$

$3x+6 = 81$

$-6 \quad -6$

$\frac{3x}{3} = \frac{75}{3}$

$\boxed{x=25}$

17) $2\sqrt{2x-6} + 8 = 4$

$-8 \quad -8$

$\frac{2\sqrt{2x-6}}{2} = \frac{-4}{2}$

$(\sqrt{2x-6})^2 = (-2)^2$

$2x-6 = 4$

$+6 \quad +6$

$\frac{2x}{2} = \frac{10}{2}$

$x=5$

Check:

$2\sqrt{2(5)-6} + 8 = 4$

$2\sqrt{10-6} + 8 = 4$

$2\sqrt{4} + 8 = 4$

$2 \cdot 2 + 8 = 4$

$4 + 8 = 4$

$12 = 4$

No!

No Solutions

18) $\sqrt{-45x^4y^7}$

$i\sqrt{45x^4y^7}$

$\sqrt{9x^4y^6} \quad \sqrt{5y}$

$3x^2y^3\sqrt{5y}$

$\boxed{3ix^2y^3\sqrt{5y}}$

19) $5\sqrt{-18} + \sqrt{-50} - \sqrt{-75}$

$5i\sqrt{18} + i\sqrt{50} - i\sqrt{75}$

$\sqrt{9}\sqrt{2}$ $\quad \sqrt{25}\sqrt{2}$ $\quad \sqrt{25}\sqrt{3}$

$5i \cdot 3\sqrt{2}$ $\quad i \cdot 5\sqrt{2}$ $\quad i\,5\sqrt{3}$

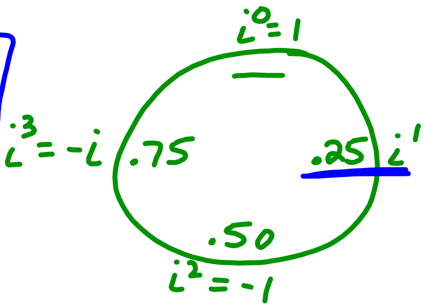
$15i\sqrt{2} + 5i\sqrt{2} - 5i\sqrt{3}$

$$\boxed{20i\sqrt{2} - 5i\sqrt{3}}$$

20) $i^{53} = \boxed{i}$

$53 \div 4 = 13.25$

4) 53 = 13.25
 4
 13
 12
 1



21) $(2-5i)(6+7i)$ $\qquad i^2 = -1$

$12 + 14i - 30i - 35i^2$

$12 - 16i - 35(-1)$

$12 - 16i + 35$

$$\boxed{47 - 16i}$$

22) a) $3x^2+5x-2$ (A, C) $AC=-6$

$3x^2+6x \mid -1x-2$ ← Grouping

$3x(x+2) \mid -1(x+2)$

$(3x-1)(x+2)$

b) $16x^4-y^8$ ← DOTS

$(4x^2-y^4)(4x^2+y^4)$

$(2x-y^2)(2x+y^2)(4x^2+y^4)$

c) $2x^2-10x-28$

$2(x^2-5x-14)$

$2(x-7)(x+2)$

d) $x^3+3x^2-4x-12$

$x^2(x+3)-4(x+3)$

$(x+3)(x^2-4)$

$(x+3)(x+2)(x-2)$

e) $8x^3+125$ SOAP

$(2x+5)((2x)^2-(2x)(5)+(5)^2)$

$(2x+5)(4x^2-10x+25)$

23) $2x^2 - 3x = 2$

$2x^2 - 3x - 2 = 0$

$a = 2$, $b = -3$, $c = -2$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)}$

$x = \frac{3 \pm \sqrt{9+16}}{4} = \frac{3 \pm \sqrt{25}}{4} = \frac{3 \pm 5}{4}$

$X = -\frac{1}{2}, 2$

$= \frac{3+5}{4} = \frac{8}{4} = 2$

AND

$= \frac{3-5}{4} = \frac{-2}{4} = -\frac{1}{2}$

24) $\frac{3x^2}{3} = \frac{6x}{3} - \frac{15}{3}$ # in front of x^2 must be 1

$x^2 = 2x - 5$

$x^2 - 2x = -5$

$x^2 - 2x + 1 = -5 + 1$

Half Square Share

$(x-1)^2 = -4$

$x - 1 = \pm\sqrt{-4}$

$x - 1 = \pm 2i$

$X = 1 \pm 2i$

25) Describe = discriminant $b^2 - 4ac$

a) $2x^2 - 3x + 4 = 0$

$(-3)^2 - 4(2)(4)$

$9 - 32$

-23 (negative) ← Imaginary Roots

b) $\frac{1}{3}x^2 - x = 6$

$\frac{1}{3}x^2 - x - 6 = 0$

$(-1)^2 - 4\left(\frac{1}{3}\right)(-6)$

$1 + 8$

9 ← Real, Rational, Unequal

Positive Perfect Square ↑

c) $x^2 - 6x = -9$

$x^2 - 6x + 9 = 0$

$(-6)^2 - 4(1)(9)$

$36 - 36$

0 ← Real, Rational, Equal

26) $x^2 - x > 6$

$-6 \quad -6$

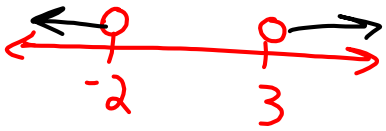
$x^2 - x - 6 > 0$

GOLI

open circles

$(x-3)(x+2)$

$x = 3 \quad x = -2$



Set Builder:

$\{x \mid x < -2 \text{ or } x > 3\}$

27) $\frac{1}{x} < 1$

$\times \frac{1}{1} = \frac{x}{x}$

$\frac{1}{x} - 1 < 0$

$\frac{1}{x} - \frac{x}{x} < 0$

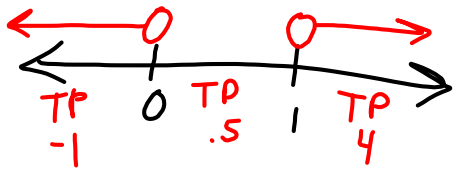
$\frac{1-x}{x} < 0$

Set Builder

$\{x \mid x < 0 \vee x > 1\}$

$1 - x = 0 \qquad x = 0$

$1 = x$ open circles



27) $x^2 - 7x \leq x$

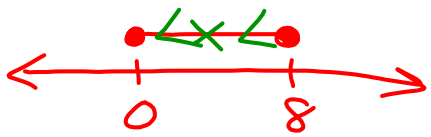
$-x \quad -x$

$x^2 - 8x \leq 0$

GOLI

$x(x-8)$

$x = 0 \quad x = 8$



$[0, 8]$

28) $\frac{9}{x-4} \geq -6$

$+6 \quad +6$

LCD $x-4$: $\frac{9}{x-4} + \frac{6(x-4)}{1(x-4)} \geq 0$

$\frac{9}{x-4} + \frac{6x-24}{x-4} \geq 0$

$\frac{-15+6x}{x-4} \geq 0$

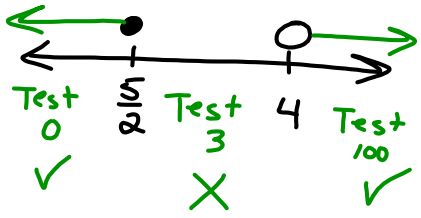
$-15+6x=0$

$\frac{6x}{6} = \frac{15}{6}$

$x = \frac{5}{2}$ closed

$x-4=0$

$x=4$ open



Interval: $(-\infty, \frac{5}{2}] \cup (4, \infty)$

29) $x^3+3x^2-9x=27$

$x^3+3x^2-9x-27=0$

$x^2(x+3)-9(x+3)=0$

$(x^2-9)(x+3)=0$

$(x+3)(x-3)(x+3)=0$

$x=-3 \quad x=3 \quad x=-3$

$x=-3, 3$

30) $x^4 - 8x^2 + 16 = 0$

$(x^2 - 4)(x^2 - 4) = 0$

$(x+2)(x-2)(x+2)(x-2) = 0$

$x = 2, -2$

31) $(x^2 + 5x - 7)(x+3) = 0$ Already Factored!

$a=1 \quad b=5 \quad c=-7$ $\qquad x = -3$

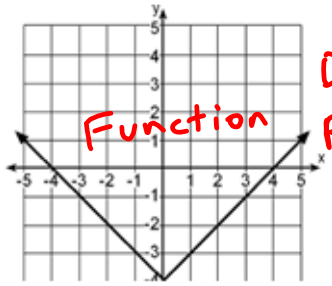
$x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(-7)}}{2(1)}$

$x = \dfrac{-5 \pm \sqrt{53}}{2}$

$x = -3, \dfrac{-5 \pm \sqrt{53}}{2}$

32. Give the domain and range for each relation. Then tell whether the relation is a function.

(a)

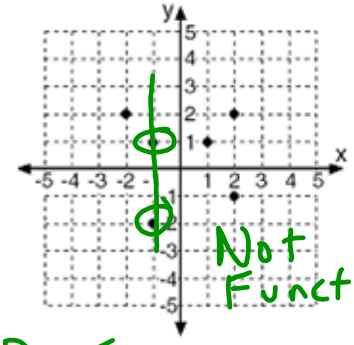


Function

D: $(-\infty, \infty)$

R: $[-4, \infty)$

(b)

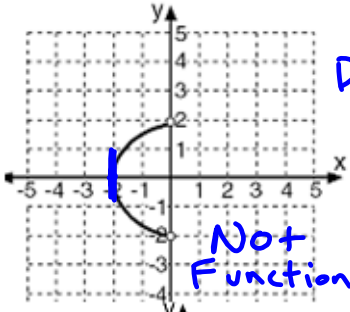


Not Function

D: $\{-2, -1, 1, 2\}$

R: $\{-2, -1, 1, 2\}$

(c)

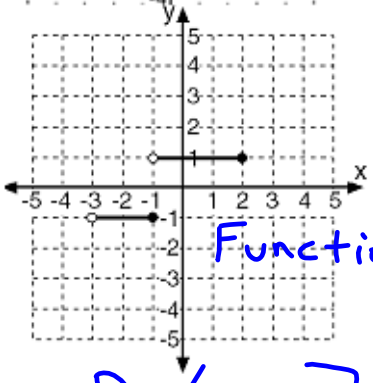


D: $[-2, 0)$

R: $(-2, 2)$

Not Function

(d)



Function

D: $(-3, 2]$

R: $\{-1, 1\}$

33) $f(x) = 4 - x^2$

$f(-2) = 4 - (-2)^2$ ($x = -2$)

$= 4 - 4$

$f(-2) = \boxed{0}$

$f(x) = -2$ ($y = -2$)

34) $3x = -10 + 4y$

$\frac{3x + 10}{4} = \frac{4y}{4}$

$\frac{3}{4}x + \frac{10}{4} = y$

$y = \frac{3}{4}x + \frac{10}{4}$ (line)

It is a function and is 1 to 1

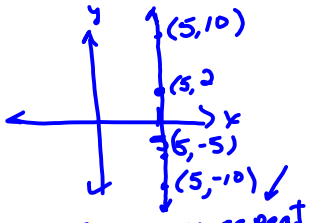
Look at the graph

35) $x = y^2 - 2y - 24$

Not a function when we have y^2.

36) $x = 5$ (vertical line)

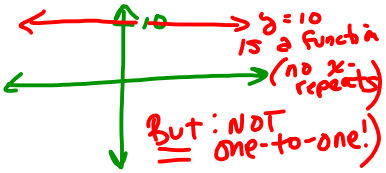
Not a function



5 is an x-repeat not a function

37) $y = 10$ (horizontal line)

It is a function but not 1 to 1



38) $y = |2x - 3|$ (absolute value)

It is a function but not 1 to 1

39) $y = x^2 - x - 6$ (Parabola)

It is a function but not 1 to 1

40) a) $\frac{x+2}{x^2-x-20}$

$x^2-x-20 \neq 0$

$(x-5)(x+4) \neq 0$

$x \neq 5 \quad x \neq -4$

$x \neq -4, 5$

b) $g(x) = \sqrt{2x-5}$

$2x-5 \geq 0$

$+5 \quad +5$

$\frac{2x}{2} \geq \frac{5}{2}$

$x \geq \frac{5}{2}$

c) $\frac{1}{\sqrt{3x-15}}$

$3x-15 > 0$

$+15 \quad +15$

$\frac{3x}{3} > \frac{15}{3}$

$x > 5$

41) $f(x) = 2\sqrt{x+3}$ $g(x) = -3x+1$

a) $f(g(1)) =$ $g(1) = -3(1)+1 = -2$

$f(-2) = 2\sqrt{-2+3} = 2\sqrt{1} = \boxed{2}$

b) $g(f(1))$

$f(1) = 2\sqrt{1+3} = 2\sqrt{4} = 4$

$g(4) = -3(4)+1 = \boxed{-11}$

c) $g(f(6))$

$f(6) = 2\sqrt{6+3} = 2\sqrt{9} = 2 \cdot 3 = 6$

$g(6) = -3(6)+1 = -18+1 = \boxed{-17}$

42) $f(x) = 4x+3$ $g(x) = \frac{x}{x+3}$

a) $f(g(x)) = f\left(\frac{x}{x+3}\right) = 4\left(\frac{x}{x+3}\right)+3$

$= \frac{4x}{x+3} + \frac{3(x+3)}{1(x+3)} = \frac{4x}{x+3} + \frac{3x+9}{x+3} = \boxed{\frac{7x+9}{x+3}}$

b) $g(f(x)) = g(4x+3) = \frac{4x+3}{4x+3+3} = \boxed{\frac{4x+3}{4x+6}}$

43) $h(x) = -x^2 - 2$

Plug in to $y=$ on calc then look at table of values

x	y
-3	-11
-2	-6
-1	-3
0	-2
1	-3
2	-6
3	-11

largest

$y = -2$

44) $f(x) = \frac{7-8x}{3} = \frac{7}{3} - \frac{8}{3}x = -\frac{8}{3}x + \frac{7}{3}$

$y = -\frac{8}{3}x + \frac{7}{3}$

It is 1 to 1 (Passes Horizontal and Vertical Line tests)

Line with a slope of $-\frac{8}{3}$ and y-int of $\frac{7}{3}$ (Look at the graph)

45) $f(x) = \{(1,2)\ (3,4)\ (5,6)\ (7,8)\}$

Domain: $\{1,3,5,7\}$

Range: $\{2,4,6,8\}$

$f^{-1}(x) = \{(2,1)(4,3)(6,5)(8,7)\}$

$f(x)$ is 1 to 1

46) a) $\frac{2x^{-2}y^{-2}}{4y^{-5}} = \frac{x^{-2}y^{3}}{2} = \boxed{\frac{y^3}{2x^2}}$ $x \neq 0$, $y \neq 0$

b) $\frac{3x^{-4}y^{5}}{(2x^{3}y^{-7})^{-2}} = \frac{3x^{-4}y^{5}}{2^{-2}x^{-6}y^{14}} = \frac{3x^2}{2^{-2}y^9} = \frac{2^2 \cdot 3x^2}{y^9} = \boxed{\frac{12x^2}{y^9}}$ $x \neq 0$, $y \neq 0$

47) $x^{-\frac{2}{5}} = \frac{x}{1}^{-\frac{2}{5}} = \frac{1}{x^{2/5}} = \boxed{\frac{1}{\sqrt[5]{x^2}}}$

✱ Power / Root

48)
$$\begin{array}{r} 2x^2-3x-3+\frac{2}{x+1} \\ x+1 \overline{)\,2x^3-x^2-6x-1} \\ -(2x^3+2x^2) \\ \hline -3x^2-6x \\ -(-3x^2-3x) \\ \hline -3x-1 \\ -(-3x-3) \\ \hline 2 \end{array}$$

$q(x) + \frac{r}{x+1}$

$= 2x^2-3x-3+\frac{2}{x+1}$

Alternate way:

$x+1=0$, $x=-1$

$$\begin{array}{r|rrrr} -1 & 2 & -1 & -6 & -1 \\ & & -2 & 3 & 3 \\ \hline & 2 & -3 & -3 & 2 \end{array}$$

$\boxed{2x^2-3x-3+\frac{2}{x+1}}$

49) $x+2=0$
$x=-2$
Plug in

$3(-2)^4+7(-2)^3+3(-2)^2-(-2)-4$
$3(16)+7(-8)+3(4)+2-4$
$48-56+12+2-4=2$

$It \neq 0$, Not a factor

50) $x+4=0$
$x=-4$

Remainder

$6(-4)^5+21(-4)^4-14(-4)^3-8(-4)^2+(-4)-6=-10$

51) $4x^4-x^3-4x^2+1x$
$\times (4x^3-x^2-4x+1)$
$\times (x^2(4x-1)-1(4x-1))$
$\times (4x-1)(x^2-1)$
$\times (4x-1)(x+1)(x-1)$

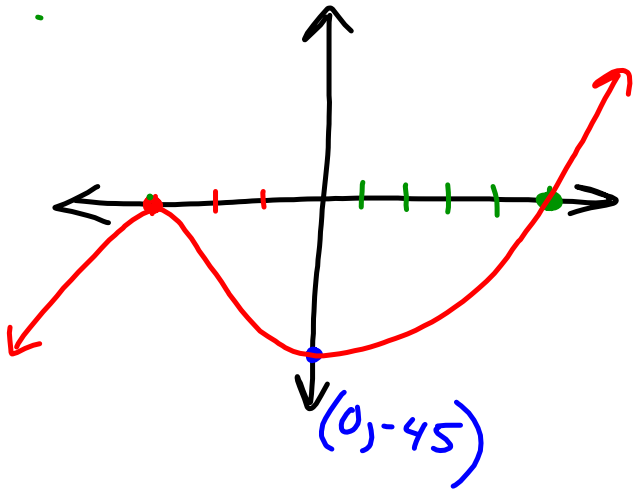
52) $P(x)=x^6+2x^4-16x^2-32$
$x^4(x^2+2)-16(x^2+2)$
$(x^2+2)(x^4-16)$
$(x^2+2)(x^2+4)(x^2-4)$
$(x^2+2)(x^2+4)(x+2)(x-2)=0$

$x^2+2=0$ | $x^2+4=0$ | $x+2=0$ | $x-2=0$
$x^2=-2$ | $x^2=-4$ | $x=-2$ | $x=2$
$x=\pm\sqrt{-2}$ | $x=\pm\sqrt{-4}$
$x=\pm i\sqrt{2}$ | $x=\pm 2i$

$x=\pm i\sqrt{2}, \pm 2i, \pm 2$

53) $f(x) = (x+3)^2(x-5)$

$x = -3$ bounce $x = 5$ cross



y-intercept:

$x = 0$ $y = (0+3)^2(0-5)$

$y = (3^2)(-5)$

$y = (9)(-5)$

$y = -45$

54) $x^2 + y^2 + 8x + 25 = 6y + 15$

$-6y \quad -25 \quad -6y \quad -25$

$x^2 + 8x + y^2 - 6y = -10$

$x^2 + 8x + \boxed{16} + y^2 - 6y + \boxed{9} = -10 + \boxed{16} + \boxed{9}$

Half Square Share

$\frac{8}{2} = 4 \quad 4^2 = 16 \qquad \frac{-6}{2} = -3 \quad (-3)^2 = 9$

$(x+4)^2 + (y-3)^2 = 15$

center: $(-4, 3)$

radius: $\sqrt{15}$

55)

	Cereal	Eggs	Total
40 and under	23	17	40
Over 40	21	(29)	50
	44	46	(90)

$$\text{Over 40 and eggs} = \frac{29}{90} = \boxed{.32}$$
Total

56) $P(\text{Even or Multiple of 3}) = P(\text{Even}) + P(\text{Mult of 3}) - P(\text{Both})$

$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

3
6

57) $P(\text{Not Prime}) = 1 - P(\text{Prime})$

$$= 1 - \frac{8}{20}$$

$$P(\text{Not}) = \frac{12}{20} = \boxed{.6}$$

58) $P(\text{Park or Beach}) = P(\text{Park}) + P(\text{Beach}) - P(\text{Both})$

$$= .38 + .54 - .22$$

$$= .70 \text{ or } \boxed{70\%}$$

59) $P(\text{Sunday and Monday}) = P(\text{Sunday}) + P(\text{Monday}) - P(\text{Sun or Mon})$

$$= .84 + .42 - .96$$

$$= .30 \text{ or } \boxed{30\%}$$

60) $P(\text{Multiple of 3 given even}) = \dfrac{P(\text{Mult. of 3 and even})}{P(\text{Even})}$

$= \dfrac{\frac{1}{6}}{\frac{3}{6}} = \boxed{\frac{1}{3}}$

61) $P(\text{No shave} \mid \text{Less than 5 hrs}) = \dfrac{P(\text{Both})}{P(\text{Less than 5})}$ (given)

$= \dfrac{.65}{.70} = .928$ or $\boxed{92.8\%}$

62) Greater than 3 / Greater than 4 / Greater than 5

$\frac{3}{6} \times \frac{2}{6} \times \frac{1}{6} = \frac{6}{216} = \boxed{\frac{1}{36}}$ (× = and, × = and)

63) 2 Rights then 2 lefts

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

OR

Left, Right, Right, Left

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

$\frac{1}{16} + \frac{1}{16} = \frac{2}{16} = \boxed{\frac{1}{8}}$

64) a) $\frac{\text{April}}{\text{Total}} = \frac{538}{947} = .568$ or **57%**

	April	May	
9th	166	64	23
10th	160	96	25
11th	124	117	24
12th	88	132	22
Total	538	409	947

b) $\frac{117+132}{947} = \frac{249}{947} = .262$ **26%**

c) $P(10^{th} \mid April) = \frac{\text{Both}}{\text{April}} = \frac{160}{538} = .297 \rightarrow$ **30%**

d) $P(April) = 57\%$ (from part a) $P(April \mid 9^{th}) = \frac{166}{230} = 72\%$

Not the same
so NOT Independent

65) $P(\text{Hot Dog or Pizza}) = P(\text{Hot Dog}) + P(\text{Pizza}) - P(\text{Both})$
$= .68 + .85 - .61$
$= .92$ or 92%

92% like Hot Dog or Pizza

$1 - .92 = .08$ **8% like neither**

Test 3
#10

zeros $-3, 1, 7$

Point $(-2, 54)$

$$y = 2(x+3)(x-1)(x-7)$$

$$54 = a(-2+3)(-2-1)(-2-7)$$

$$= a(1)(-3)(-9)$$

$$54 = a(27)$$

$$a = 2$$

12)

$$x-2 \overline{)\, x^3+2x^2+0x-7}$$

$$x^2+4x+8+\frac{9}{x-2}$$

$$+(-x^3+2x^2)$$

$$4x^2+0x$$

$$+(-4x^2+8x)$$

$$8x-7$$

$$+(-8x+16)$$

$$9$$

$$(x-h)^2+(y-k)^2=r^2$$

13) $x^2+4x+\boxed{4}+y^2-6y+\boxed{9}=7+\boxed{4}+\boxed{9}$

$$(x+2)^2+(y-3)^2=20$$

C: $-2, 3$

$r=\sqrt{20}$

14) $h(x) = 10 - x^2$

$h(f(2)) =$

$f(2) = 8$

$10 - 8^2 =$

$10 - 64 = -54$

7) $x^3 + 10x^2 + x - 120$

$x + 3 = -3$

$x - 2 = 2$

$x - 4 = 4$

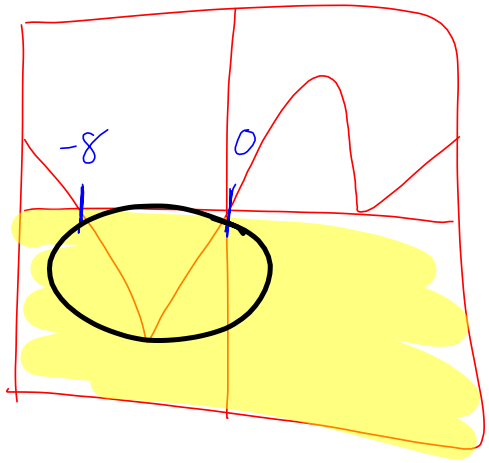
$x + 5 = -5$

14) b) $f(x) < 0$

$y < 0$

$(-8, 0)$ or

$-8 < x < 0$



9) $P(x) = x^5 - 6x^3 - 27x$

$x(x^4 - 6x^2 - 27)$

$(x^2 - 9)(x^2 + 3)$

$x(x-3)(x+3)(x^2+3)$

$x = 0 \quad 3 \quad -3$

$x^2 + 3 = 0$